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Dynamic simulation of a flexible pavement layers considering shakedown effects and soil-asphalt interaction

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ABSTRACT

This research evaluates the effects of shakedown and soil asphalt interaction on the dynamic simulation of flexible layered pavement structure. In a newly developed assessment methodology the shakedown concept is implemented for the granular layer under asphalt concrete. Under this assessment granular material behaviour changes from plastic to elastic as a function of the number of loading cycles. Interactional forces are considered through contact elements placed between the base and the asphalt layer. A nonlinear stress dependent model in the elastic domain and the Mohr–Coulomb constitutive model in the plastic domain are utilised to simulate the behaviour of granular layers and the results of the simulation are validated with previously published literature. One static simulation and three different dynamic simulations under cyclic Haversine loading of a single tyre dual axle are then compared with each other. The first dynamic simulation assumes Mohr–Coulomb plasticity, the second simulation considers the shakedown effects and the third simulation calculates soil asphalt interactional forces.

Introduction

In flexible pavement engineering the significant role of the granular layers in rutting failure is well known. The investigation of the contribution of granular materials in rutting and surface deflection in accelerated pavement tests has been previously examined (Little, 1993; Pidwerbesky, 1996; Arnold et al., 2001; Korkiala-Tanttu et al., 2003). Based on this research, the layers containing granular materials have been considered responsible for 30–70% of the rutting deflection occurring at the surface of flexible pavements. The rutting itself is governed by the permanent deformation and the residual plastic strains induced inside the granular body in each loading cycle. This is why the investigation of unbound granular materials (UGM) and their reaction to the cycles of traffic loading is considered critical.

The behaviour of granular materials can be divided into two parts: elastic range and plastic range performance. Whilst induced stress remains in the elastic domain, material behaviour can be modelled as linear elastic or nonlinear elastic. For UGM it is more realistic to consider a stress dependent elastic modulus where elastic modulus (or resilient modulus) is not constant during the increments of loading. Such an approach has been widely used in previous research (Cho et al., 1996; Kim and Tutumluer, 2006; Kim et al., 2009; Lee et al., 2009; Cortes et al., 2012; Ghadimi et al., 2013a,b,c). There is a variety of nonlinear constitutive models developed especially for granular material assessment in flexible pavement. Nonlinearity of granular material can be considered through the dependency of the resilient modulus to the strain tensor (Hjelmstad and Taciroglu, 2000; Taciroglu and Hjelmstad, 2002) to the bulk stress (Seed et al., 1962; Hicks and Monismith, 1971) or to the stress invariants representing deviatoric and bulk stress conditions (Uzan, 1985; Witczak and Uzan, 1988). Amongst

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forms of nonlinearity, the latter has found more interest amongst flexible pavement researchers where UGM behaviour is largely defined as a function of deviatoric and bulk stress conditions simultaneously.

If no cap is defined for the UGM, the stress–strain path will remain the same in the loading and unloading scenarios. However, in reality there is some residual strain in the UGM layers after each loading cycle which necessitates the usage of elastoplastic constitutive models in the numerical simulation of the granular layers. There are various constitutive models to represent plastic behaviour of UGM. Among them Drucker–Prager criterion and Mohr–Coulomb criterion have been used commonly in the simulation of granular materials in flexible pavement (Zaghloul and White, 1993; Shen and Kirkner, 2001; Saad et al., 2005; Ghadimi et al., 2014). The two criteria are compared in Fig. 1 where $\sigma1$, $\sigma2$ and $\sigma3$ are the three principle stresses. The main disadvantage of the Moher–Coulomb criterion when compare to the Drucker–Prager criterion is in the numerical simulation where the failure surface is not a smooth differentiable surface and may cause some inconsistency during incremental analysis. However, Clausen et al. (2007) introduced a method to include Mohr–Coulomb criterion in the ABAQUS programme through a piece-wise function and return algorithm and this same methodology is employed in this study.

The goal of numerical simulation of pavement structure is to evaluate the mechanical responses to a given traffic load. The usage of a general purpose finite element programme made this approach more feasible in the field of pavement engineering. ABAQUS programme is used by many researchers who have conducted numerical simulations (Zaghloul and White, 1993; Uddin et al., 1994; Kim and Tutumluer, 2006, 2010; Cortes et al., 2012; Ghadimi et al., 2013a,b,c, 2014). This software programme permits researchers to define new constitutive models for the materials in the simulation of different types of loading and boundary conditions. The accuracy of finite element simulation can be adjusted through geometrical dimension (two dimensional or three dimensional), assumed materials behaviour (linear, nonlinear, elastic or plastic), boundary conditions (rollers, clamped or infinite elements) and loading conditions (static or dynamic).

Static loading provides the advantage of simplifying computing efforts when a complex material model is used.

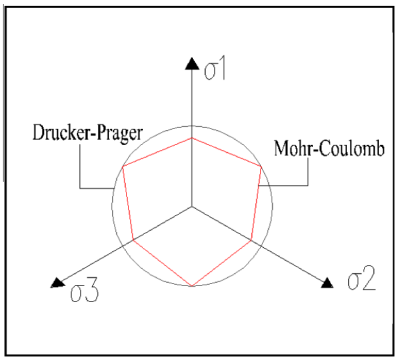

Fig. 1. Yield criteria used for granular materials.

Many researchers have investigated the effects of material behaviour in layered flexible pavement under static loading (Cho et al., 1996; Myers et al., 2001; Holanda et al., 2006; Kim and Tutumluer, 2006; Kim et al., 2009; Ghadimi et al., 2013a,b,c). However, the assumption of static loading overlooks some important aspects. For example the effects of damping and body forces are ignored in static analysis. This may result in the calculation of larger deformation in static loading than actual experiments. Such effects have been reported in previous studies (Zaghloul and White, 1993; Uddin et al., 1994; Saad et al., 2005). Furthermore, under the assumption of static conditions, the effects of the unloading path cannot be investigated. This is especially important if material behaviour is dependent on loading path (elastoplastic).

The numerical analysis of pavement layers subjected to dynamic loading provides more accurate insight into the actual mechanical responses of the pavement structure. This, however, necessitates more complex computation. One of the earliest studies considering dynamic loading in numerical simulation was conducted by Zaghloul and White (1993) where ABAQUS software was employed to simulate the responses of layered flexible pavement. After this study, dynamic simulation has continued to grow amongst pavement researchers (Uddin and Pan, 1995; Saad et al., 2005; Bodhinayake, 2008; Al-Qadi et al., 2010; Ghadimi et al., 2014). In dynamic simulation major points which need to be considered include the effects of boundary conditions, layer interactions, the wave function of loading pluses and any materials change due to loading cycles. Boundary conditions can be simulated as rollers and pins (Zaghloul and White, 1993; Uddin and Pan, 1995; Bodhinayake, 2008) or infinite elements (Al-Qadi et al., 2010; Ghadimi et al., 2014). The latter may reduce the effects of reflective stress waves in the simulated medium. In terms of layers interaction, there are only a few studies that consider this effect in flexible pavements (Pan et al., 1994; Baek et al., 2010; Ozer et al., 2012), but have been considered in the other similar layered structure assessment (Mishra and Tutumluer, 2012). Loading pulses from tyres may also be simulated to have a triangular (Saad et al., 2005; Bodhinayake, 2008), trapezoidal (Zaghloul and White, 1993), sinusoidal (Al-Qadi et al., 2010) or Haversine (Ghadimi et al., 2014) function varying in the time span of the stress pulse. The behaviour of the granular layers is assumed to be modified in each of the loading cycles.

In the case that applied load exceeds yield criterion of the UGM, the response of the granular material may change according to the cycles of loading. To study this phenomenon the shakedown theory was developed (Melan, 1938; Zarka and Casier, 1979). This theory was applied through limit analysis in flexible pavement layers by Sharp and Booker, 1984, 1985. Other approaches to calculate the analytical shakedown limit in a layered flexible pavement subjected to tyre pressure can be via the upper-bound solution (Collins and Boulbibane, 1998) or the lower-bound solution (Yu and Hossain, 1998). Researchers have also tried to investigate the shakedown limits of UGM in laboratory experiments (Barksdale, 1972; Siripun et al., 2010; Cerni et al., 2012). The experimental approaches usually are conducted through long-run triaxial dynamic loading tests performed in various stress ratios for a large number of loading cycles. In recent years, attempts have been made to consider the effects of shakedown in pseudo-dynamic finite element simulation of pavement structure (Habiballah and Chazallon, 2005; Allou et al., 2009; Chazallon et al., 2009; Ghadimi et al., 2014). However, there is still a need to implement a shakedown model in a dynamic analysis. In the current study three dimensional models has been constructed and the effect of shakedown for granular materials has been considered under dynamic analysis.

This study aims to investigate the effects of dynamic loading on granular materials used in the base of the flexible pavement structures considering soil-asphalt interaction (SAI) and shakedown effects. A series of three dimensional dynamic simulations were conducted on models constructed in ABAQUS software. In these simulations the asphalt layer was assumed to be linear elastic whilst the base and the subgrade had nonlinear elastoplastic properties. The outcome of these dynamic simulations highlights the important effects of shakedown and SAI in critical design parameters of the pavement structure.

Mechanical responses of granular materials to cyclic loading

The mechanical responses of UGM subjected to cyclic loading in the field of pavement engineering can be studied in three stages: (a) in the elastic domain and before the yield, (b) in the yield state and (c) in the growth of the plastic zone after the yield.

In the elastic domain, the stress–strain relationship of the materials is assumed to be the same in loading and unloading. This simple relationship can be defined when stress is a linear function of strain according to Hook's law. In three dimensional continuum mechanics it can be written as follow:

$$\begin{Bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{33}\\\sigma_{12}\\\sigma_{23}\\\sigma_{31}\end{Bmatrix}=\frac{E}{(1+v)(1-2v)}\times\begin{pmatrix}1-v & v & v & 0 & 0 & 0\\ & 1-v & v & 0 & 0 & 0\\ & & 1-v & 0 & 0 & 0\\ & & & \frac{1-2v}{2} & 0 & 0\\ & \text{Sym.} & & & \frac{1-2v}{2} & 0\\ & & & & & \frac{1-2v}{2}\end{pmatrix}\begin{Bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\varepsilon_{33}\\\varepsilon_{12}\\\varepsilon_{23}\\\varepsilon_{31}\end{Bmatrix} \quad (1)$$

where σ_{ij} is the stress components in three different orthogonal directions (x, y and z), ε_{ij} is the strain components in three different orthogonal directions (x, y and z), E is the elastic modulus and v represents the Poisson ratio.

In flexible pavement, the concept of the resilient modulus (M_R) replaces the elastic modulus. Resilient modulus is the elastic modulus based on the recoverable strain. However, the linear relationship between stress and strain in UGM may be another simplification which may not represent the materials actual behaviour. As mentioned previously, many different relationships have been introduced to represent the nonlinear relationship of stress and strain in UGM (Witczak and Uzan, 1988). Eq. (2) determines the mathematical form of this model:

$$M_R = K_1 P_0 \left(\frac{I_1}{P_0}\right)^{k_2}\left(\frac{\tau_{oct}}{P_0}\right)^{k_3} \quad (2)$$

$$I_1 = \sigma_{11}+\sigma_{22}+\sigma_{33}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2]}$$

In this equation the resilient modulus of granular (M_R) is related to the first invariant of the stress tensor (I_1) and the octahedral shear stress (τ_{oct}). P_0 is the unit pressure, K_1, k_2 and k_3 are the materials constants determined from laboratory tests. Many studies have employed the model to investigate UGM behaviour (Cho et al., 1996; Kim and Tutumluer, 2006; Bodhinayake, 2008; Kim et al., 2009; Ghadimi et al., 2013) as the model considers the contribution of both hydrostatic and deviatoric components of stress in the determination of the resilient modulus.

The behaviour of UGM cannot be defined by the elastic modulus. If the combination of stress components induced in the UGM exceeds a certain limit then it is assumed that the material has reached the plastic limit. This limit is based on components of stress to indicate the maximum capacity of stress that can be sustained by materials before plasticity. A well-known Mohr–Coulomb criterion is one of the earliest developed flow rules where the plastic cap of granular materials is stated in terms of the friction and cohesion of granular components. The Mohr–Coulomb flow rule is indicated in Eq. (3) (Yu, 2006):

$$\begin{aligned}&\sqrt{J_2}-\tfrac{m(\theta_l,\varphi)}{3}I_1-m(\theta_l,\varphi)c\cos\varphi=0\\&m(\theta_l,\varphi)=\tfrac{\sqrt{3}}{(\sqrt{3}\cos\theta_l+\sin\theta_l\sin\varphi)}\end{aligned} \quad (3)$$

$$J_2=\frac{1}{6}\left[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2\right]+\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2$$

$$\theta_l=\tan^{-1}\left[\frac{\sqrt{3}}{3}\left(\frac{2\sigma_3-\sigma_1-\sigma_2}{\sigma_1-\sigma_2}\right)\right]$$

In the equation θ_l is Lode's angle, c is cohesion and φ is internal friction of materials.

Although the introduction of yield limit provides a more realistic estimation of UGM responses to loads/pressures, it cannot account for any change in the materials due to the cycles of loading. In other words, applying the same magnitude of loading on an element of UGM will result in the same magnitude of strain. However, this is not always a true indication of UGM responses to cyclic loading. In fact there are cases in which UGM demonstrates negligible

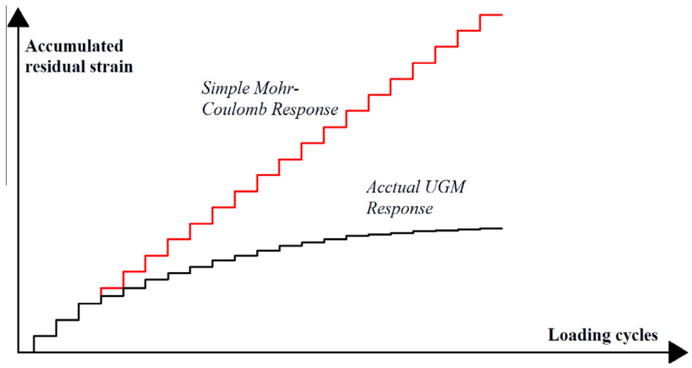

Fig. 2. Response of granular materials to cyclic loading.

plastic strain after sufficient cycles of loading have been applied. Fig. 2 illustrates these two different responses.

One of the useful approaches to deal with this phenomenon is the application of the shakedown theory developed by Melan (1938). According to Boulbibane and Weichert (1997) if repeated loading on the granular induces stress beyond the yield surface, three different responses may be observed. Fig. 3 illustrates these responses schematically.

In case A the residual strain in the materials increases almost without limit. This so-called "ratcheting" state is close to what can be predicted applying simple Mohr–Coulomb criterion to a cyclic loading. In the responses like case B, residual strain in the materials grows to some extent, but at some stage the growth is stopped and further cyclic loading produces closed hysteresis loops of stress–strain. Finally in case C the growth of residual strain is practically diminishes when sufficient loading cycles are applied. Case B and case C are cases of plastic and elastic shakedown respectively (Boulbibane and Weichert, 1997). It can therefore be concluded that there are conditions in cyclic loading of UGM in which the growth of plastic strain is limited. From an engineering point of view, a design including pavement layers cannot allow a loading large enough to produce case A conditions. However, cases B and C may be tolerated especially in flexible pavement structures where a restricted amount of permanent deformation will not reduce functionality.

The shakedown effects especially have a role in the dynamic simulation of pavement structure where a large number of loading cycles is considered. It can be understood from Fig. 2 that if shakedown effects are not considered, a gradual growth of residual strain is calculated to be limitless. In numerical simulation this leads to divergences in calculation. However, shakedown may result in cases where stable behaviour of materials is achieved after the initial cycles of loading. Therefore considering shakedown effects may assist the numerical simulation to converge.

Characteristics of the finite element model

In this study finite element methodology (FEM) is employed to conduct numerical simulations of the dynamic loading on the layered structure of flexible pavement. The preliminary simulations were conducted using proper mesh density and distance of boundary conditions. Three dimensional modelling of a dual tyre single axle load producing a uniformly distributed pressure of 750 kPa over a rectangular area of the tyre contact was considered. The nonlinear elastoplastic behaviour of UGM is considered based on Witczak–Uzan and Mohr–Coulomb model as explained in the previous sections. In all the simulations asphalt layers were assumed to behave linear elastically. Special attention in this simulation is given to the base layer where two cases considering shakedown effects are investigated. Specifically developed material behaviour was implemented through user-defined material subroutine (UMAT) in the implicit dynamic analysis in ABAQUS software.

Geometric properties of the model

Three possible models can be selected: (a) 2-dimensional plane strain model, (b) 2-dimensional axisymmetric model or (c) 3-dimensional models. Whilst there are still debates amongst researchers about the

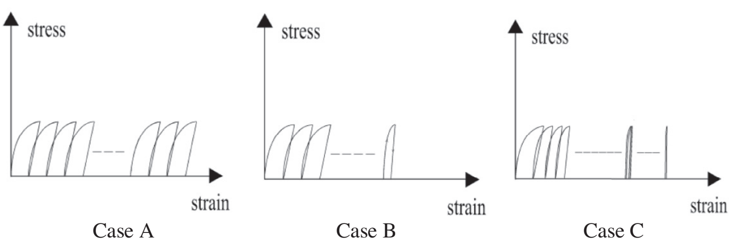

Fig. 3. Three different hysteresis loops of UGM in response to cyclic loading.

benefit-cost ratio of three dimensional modelling, it is a generally accepted idea that three dimensional analyses provide more accurate results (Cho et al., 1996; Saad et al., 2005; Ghadimi et al., 2013a,b,c). Three dimensional modelling enables simulation of different tyre foot prints on an asphalt layer as well as different tyre-axle combinations. However, the computation time of the models increases by an order of 3. In this simulation three dimensional models were constructed in ABAQUS software in order to consider complete single axle dual tyre loading effects.

In this analysis width, length and height of the model is selected to be greater than those recommended in the literature (Kim et al., 2009) to minimise the effect of boundary conditions on the results. The mesh distribution is selected to be finer, closer to the tyre loading and coarser close to the model boundaries (Fig. 4).

Conducting the mesh sensitivity analysis, the final model was constructed from 59,392 elements and 64,185 nodes. The model has two classes of elements to represent soil medium (C3D8R) and the infinite boundaries of the model (CIN3D8).

Layer properties

Since the research is focused on UGM, the asphalt layers are simulated with linear elastic properties. This can also be justified knowing that asphalt stiffness is far larger than those for UGM.

To study the effects of layer interaction in dynamic analysis, the interface element was defined between the asphalt and granular layers. The interface elements have Coulomb frictional behaviour where the contact stress is distributed according to Eq. (4) where the pressure is calculated based on the virtual distance of two surfaces.

$$\begin{cases} P = 0 & \text{if} \quad t < 0 \text{ (open)} \\ t = 0 & \text{if} \quad p > 0 \text{ (closed)} \end{cases} \qquad \delta W = \delta p dt + dp\delta t \tag{4}$$

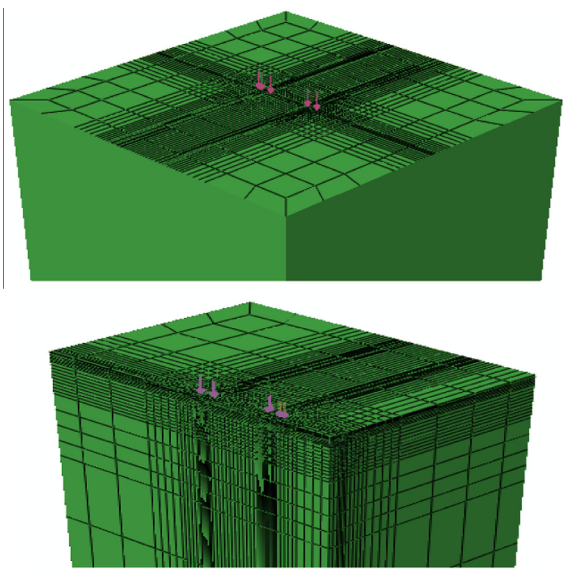

Fig. 4. Constructed mesh for dynamic analysis.

In this equation W is the virtual work, t is the virtual distance between surfaces and p is the distributed stress.

Loading and boundary conditions

The contact pressure of the tyres is uniformly distributed on an equivalent rectangular area (Huang, 2004). The analyses were conducted in the simulation of a single axle dual tyre based on AUSTROADS (2004). The loading condition is schematically represented in Fig. 5.

The Haversine function is selected to adopt the variation in loading pressure over time. It should be noted that the stress pulses are not continuous and there is a rest gap between each pulse. Fig. 5 illustrates the amplitude of contact stress in time where loading occurred at 0.1 s and a rest period spanned 0.9 s.

The model is clamped at the bottom which prevents any degree of freedom for all nodes at the bottom surface. However, sides of the medium were modelled through infinite elements which can reduce reflection of pressure waves. These elements apply damping to induced stress according to Eq. (5):

$$\begin{cases} \sigma_{xx} = -d_p v_x \\ \sigma_{xy} = -d_s v_y \\ \sigma_{xz} = -d_s v_z \end{cases} \tag{5}$$

where d_p and d_s denote the ratio of damping to pressure waves and shear waves and v is the velocity.

Constitutive modelling of materials

Asphalt

The contribution of the asphalt layer in the layered pavement structure is to resist against the force induced by tyre pressure. The stiffness and strength of this layer is significantly higher than other granular layers. It is also worth mentioning that the focus of the current study is to investigate the effect of shakedown on the granular layers. Therefore for the purpose of simplicity and to avoid unnecessary complexity in an understanding of the results, asphalt layers are modelled as a linear elastic material. This assumption is also used by other researchers (Saad et al., 2005) to numerically model the pavement response.

Base

The nonlinearity of the base materials is also investigated in this research. The nonlinear behaviour of the materials is considered in both the elastic and the plastic domains. Nonlinear elasticity is implemented through Witczak and Uzan's (1988) stress dependent model described in Eq. (2). The initial yield surface to consider plasticity is defined by the Mohr–Coulomb in Eq. (3). Mohr–Coulomb criteria can produce some difficulties at vertices (Saad et al., 2005). However, using a piece wise function in the simulation as introduced by Clausen et al. (2007) the Mohr–Coulomb was integrated into the analysis.

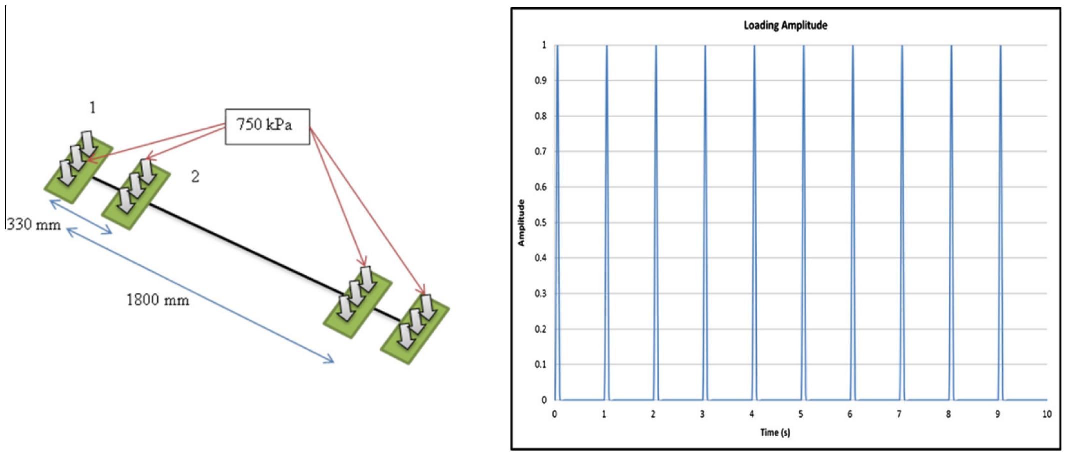

Fig. 5. Single axle dual tyre loading and Haversine pulses in time.

Shakedown behaviour of the material to simulate the UGM is shown in Fig. 2. When loading is applied three cases of stress field occurs as follows:

$$\begin{cases} g(\sigma_{ij}^k) > L_1 \rightarrow \varepsilon_{ij}^p(t) = f_1(N)\varepsilon_{ij}^p(0) \\ L_2 \leqslant g(\sigma_{ij}^k) \leqslant L_1 \rightarrow \varepsilon_{ij}^p(t) = f_2(N)\varepsilon_{ij}^p(0) \\ g(\sigma_{ij}^k) < L_2 \rightarrow \varepsilon_{ij}^p(t) = f_3(N)\varepsilon_{ij}^p(0) \end{cases} \tag{6}$$

In this equation N is the number of load cycles and f_1, f_2 and f_3 are the experimentally indicated function of N. L_1, L_2 and L_3 are limits applied to stress fields indicated as $g\,(\sigma)$. Superscript k indicates the number of increments and subscript i and j are indicators of the co-ordinates. In this equation $\varepsilon^p(0)$ is the initially induced plastic strain.

Without considering the effects of shakedown, plastic strain is solely determined by the stress field as a function of the loading magnitude. In repetitive constant loading, the stress field induced in each cycle does not vary significantly and almost constant values of plastic strain are produced in each of the cycles. However, if the stress field satisfies the shakedown conditions (determined by L limits in Eq. (6) the behaviour of the material gradually moves from elastoplastic to purely elastic (hypothetically when the number of cycles tends to infinity). This case is mathematically presented in Eq. (7):

$$\sigma^{\mathrm{N}} = \mathrm{C}^{\mathrm{N}}\varepsilon^{\mathrm{N}}$$
$$\begin{cases} \text{if } N = 1 \rightarrow \sigma^N = (C^N)^{ep}(\varepsilon^N)^{ep} \\ \text{if } N \rightarrow \infty \rightarrow \sigma^N = (C^N)^{e}(\varepsilon^N)^{e} \end{cases} \tag{7}$$

where N is the number of cycles and C is the material stiffness matrix. Superscript e indicates elastic behaviour whilst superscript ep indicates elastoplastic behaviour.

Eq. (6) can be differentiated in terms of time as follows:

$$\frac{\partial \varepsilon^p}{\partial t} = \frac{\partial f(N)}{\partial N}\varepsilon_0^p \tag{8}$$

And from Eq. (7) it can be written

$$\begin{cases} \text{if } N = 1 \rightarrow \frac{\partial f_i(1)}{\partial N} = 1 \\ \text{if } N \rightarrow \infty \rightarrow \frac{\partial f_i(N)}{\partial N} = 0 \end{cases} \tag{9}$$

Substituting Eqs. (6) and (7) into (9) the material stiffness matrix can be modified as follows:

$$\sigma^N = \left(\frac{\partial f(N)}{\partial N}\right)(C^N)^{ep}(\varepsilon^N)^{ep} + \left(1 - \frac{\partial f(N)}{\partial N}\right)(C^N)^{e}(\varepsilon^N)^{e} \tag{10}$$

The change of behaviour according to the number of the loading cycles can be simulated through Eq. (10).

Subgrade

The subgrade of flexible pavements is usually constructed from lower quality materials. The subgrade may be variable from clayey soil to fine sand. The current Mohr–Coulomb elastoplastic model is applied to simulate the behaviour of soil in the subgrade layers. The soil properties are selected to represent medium quality subgrade. In this layer the shakedown effects are not considered. The numbers of static simulations indicates that stress passed from the base layer to the subgrade will fall below the elastic limit of the shakedown and therefore the inclusion of a subroutine will only increase computation time.

Effect of dynamic analysis

Static analysis vs dynamic

The first step of the analysis was to verify the simulation in the nonlinear elastic domain for static loading. Static analysis also helps to understand the effect of dynamic loading more inclusively. The nonlinear elastic constitutive model was implemented in the same approach introduced by Kim et al. (2009). The same model was then reconstructed to verify the developed UMAT for nonlinear elastic assessment. The model used an axisymmetric medium with linear asphalt layer and a nonlinear base layer. The properties of the nonlinear materials were considered through the Witczak and Uzan (1988) stress dependent model where $K1 = 4.1$ MPa, $k2 = 0.64$ and $k3 = 0.065$.

The results of four critical responses for the flexible pavement calculated in the current study were then compared by the responses reported by Kim et al. (2009), and are presented in Table 1.

As it can be seen, the results of the simulation in the current study are very similar to those reported by Kim et al. (2009).

In the next part of the analysis, the three dimensional model described in the Section Geometric properties of the model was subjected to both static and dynamic loading as stated in Section Loading and boundary conditions. In this step, the weight of the soil was considered. The modelled soil body had initial at rest stress before being subjected to the tyre pressure.

For this analysis material properties are listed in Table 2. The granular base is simulated to reflect nonlinear elastic (Witczak and Uzan, 1988) and also plastic Mohr–Coulomb responses. A 10% Rayleigh damping was included to reduce the resonance effects in the modelled area. The density of the materials used were selected according to typical materials detailed in AUSTROADS (2004) specifications.

The calculated results of the static and dynamic analysis in terms of vertical deformation are illustrated in Fig. 6. It can be seen that maximum static vertical deformation occurred beneath the inner tyre and reached 1.03E−3 m (1.03 mm).

The contours of vertical deformation in the dynamic analysis are similar to those in static analysis but have smaller values. Table 3 presents the calculated values of the four critical design parameters for both of the analyses. The values are presented for the axis passing through the edge of the outer tyre. The smaller deformation in the dynamic analysis has been reported in the literature (Zaghloul and White, 1993; Saad et al., 2005) and same is confirmed in this study. Another observation can be made by comparison between the results of 1000th and final cycle of the dynamic analysis where a significant increase of critical responses can be reported in the final cycle. For example an increase of 400% is calculated for the vertical strain at the top subgrade (from 217E−6 at the 1000th cycle to 870E−6 at the end of analysis). This increase can be explained by the effect of the Mohr–Coulomb constitutive models and the accumulation of constant residual plastic strain.

Fig. 7 presents the sequence of the vertical deformation in the 1000th cycle of dynamic analysis for the outer tyre in both longitudinal and transverse directions. The longitudinal direction is the direction of the travel whilst the transversal direction is the direction of the axle of the vehicle. The loading is assumed to be a stress pulse of duration 0.1 s whilst there is a 0.9 s rest period between each cycle of loading. From Fig. 7 it can be seen that there is permanent surface deformation after completion of the rest period (at the end of each 1 s cycle). This can be attributed to the elastoplastic constitutive model which causes plastic strain in the granular layers.

It can be observed that the vertical deformation gradually increases to reach to a final value of 4E−4 m (0.4 mm) in 0.1 s at the end of pulse loading phase. Then this deformation reduced to1.5E−5 m in 0.9 s in the rest period. This residual permanent deformation is a result of accumulated permanent deformation in the flexible pavement structure.

In Fig. 7 the maximum vertical deformation is calculated to be beneath the inner tyres of a single axle dual tyre loading. It can be said that the critical loading tyre for design purposes is therefore the inner tyre.

A comparison between the loading and rest period in the transverse direction indicates the clear effects of the tyres in the pulse loading phase, whilst the rest period does not result in such effects.

Long term dynamic analysis

The long term dynamic response of granular materials is studied in this section. The dynamic analysis is conducted for 100,000 cycles of tyre loading. If an elastoplastic

Table 1
Verification of the nonlinear elastic model.

Critical response	Current research (linear)	Kim–Tutumluer (linear)	Current research (nonlinear)	Kim–Tutumluer (nonlinear)
Surface deflection (mm)	0.930	0.930	1.276	1.240
Tensile strain bottom of AC	251E−6	227E−6	312 E−6	267E−6
Compressive strain top of SG	921E−6	933E−6	1170E−6	1203E−6
Compressive stress (kPa) top of SG	40	41	54	Not presented

Table 2
Material properties in dynamic analysis.

Properties	Layer		
	Asphalt (AC)	Base	Subgrade
Thickness	100 mm	200 mm	2000 mm
E	2800 (MPa)	$K1 = 332$ (MPa), $k2 = 0.08$, $k3 = 0.2$	50 (MPa)
v	0.35	0.4	0.45
φ	0	30°	20°
C	0	0.01 (MPa)	0.01 (MPa)
ψ	0	15°	10°
Density (kg/m^3)	2400	1800	1700
Rayleigh damping	10%	10%	10%

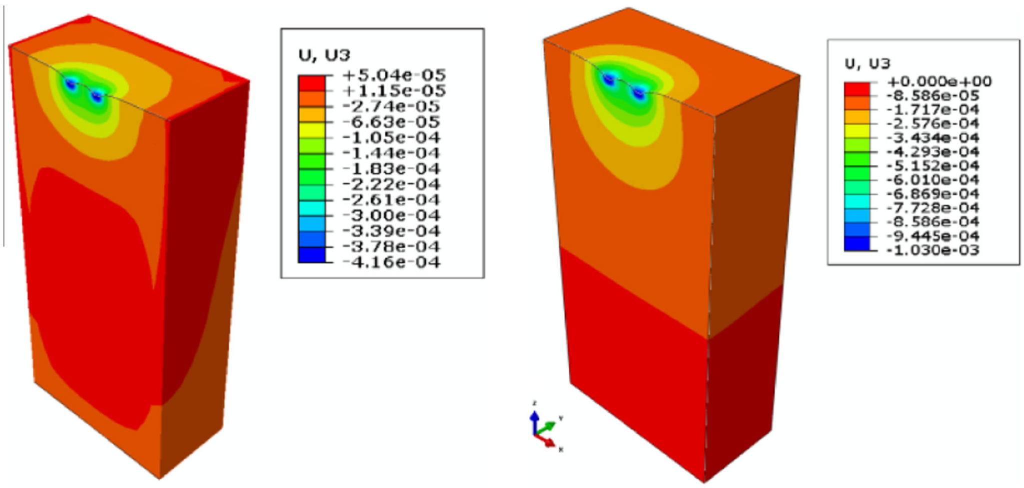

Fig. 6. Vertical deformations in dynamic (left) and static (right) analysis.

Table 3
Comparison of the critical responses between static and dynamic simulation.

Analysis	Vertical deflection (top of AC)	Tensile strain (bottom of AC)	Compressive strain (top of SG)	Compressive stress (top of SG)
Static	1.01 mm	231E−6	940E−6	50.70 kPa
Dynamic-1000th cycle	0.4 mm	150E−6	217E−6	19.11 kPa
Dynamic-final cycle (78,979 cycles)	1.0 mm	1050E−6	870E−6	57.70 kPa

constitutive model is assumed, the effect of long term dynamic loading on the residual displacement and accumulated plastic strain can be significant. Fig. 8 presents the surface deflection underneath the exterior tyre during the loading cycles.

The equivalent plastic strain is a scalar value that can be a representative of plastic strain tensor in each increment. Equivalent plastic strain in each increment is defined as follow:

$$\mathrm{PEEQ} = \sqrt{\frac{2}{3} PE_{ij} : PE_{ij}} \tag{11}$$

where PEEQ is equivalent plastic strain and PE_{ij} are the components of plastic strain tensor at the given increment of analysis.

It can be seen from Fig. 8 that the initial vertical deflection of 4E−4 m increased linearly to 10E−4 m at the end of the simulation where a numerical divergence occurred due to the unacceptable large strain.

The vertical strain and equivalent plastic strain growth during the loading cycles are presented in the Fig. 8. Whilst the vertical strain at the first 1000 cycles of loading reached to −0.000876 it gradually increased to −0.0037 as shown in Fig. 8. The increase of vertical strain is almost linear which implies the accumulation of the same amount of plastic energy in every cycle of loading.

This can be confirmed when the trend of the equivalent plastic strain (SDV4) is studied (Fig. 8). Due to a linear increase in equivalent plastic strain, the simulation exceeded the range of the small strain assumption and therefore diverged. Fig. 9 shows the sequences of development in the contours of equivalent plastic strain in the base layer. It can be seen that the granular materials yielded beneath the loading tyres.

The hysteresis loops of the stress–strain at the top of the base layer under the outer tyre are presented in Fig. 10. It can be seen that the growth of the hysteresis loop is continuous across the total loading time. In the final cycles, the hysteresis loops create the same area (approximately) in the loading cycles (Fig. 10) which mean the constant accumulation of energy in each cycle.

Effect of shakedown

Simulation of triaxial sample

The constitutive model to reflect shakedown effects is included in the simulation of the triaxial tests. An axisymmetric model to represent a standard triaxial cylinder is simulated in ABAQUS. The results of the PRG group at Curtin University (Siripun et al., 2010) were used for the purpose of verification. In this model the material properties stated in Table 1 by Siripun et al. (2010) is also adopted in the simulation. The previously described method to include the shakedown effect is integrated in the Mohr–Coulomb constitutive model. Fig. 11 illustrates the calculated results of accumulated plastic strain after 600,000 cycles. A similar trend of increase in plastic strain can be seen in this figure whilst the maximum difference is less than 3%.

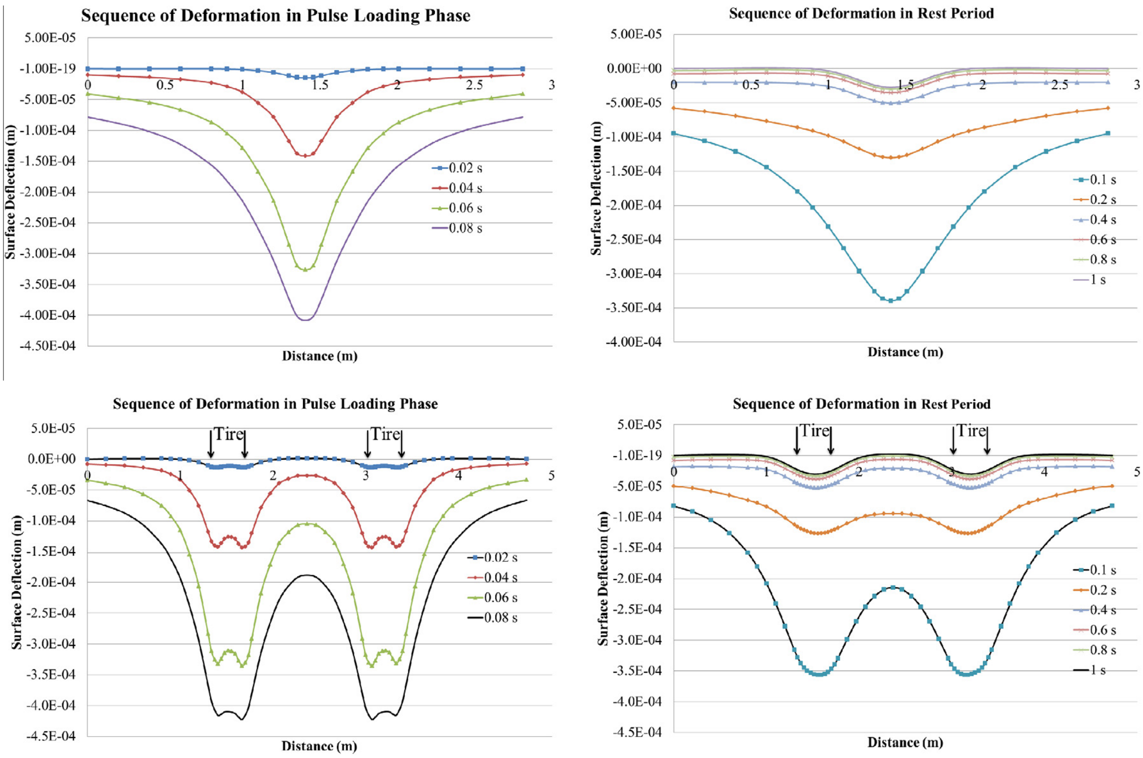

Fig. 7. Longitudinal (up) and transverse (bottom) deformation without the shakedown.

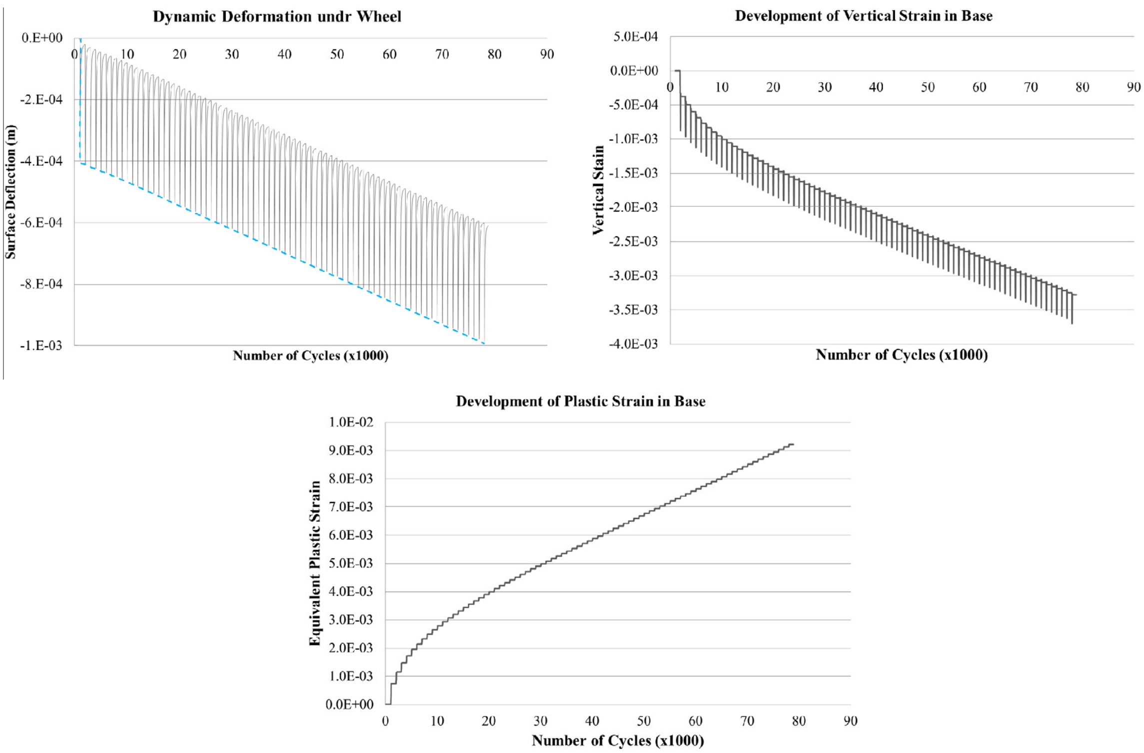

Fig. 8. Mechanical responses of the base beneath the tyre without shakedown.

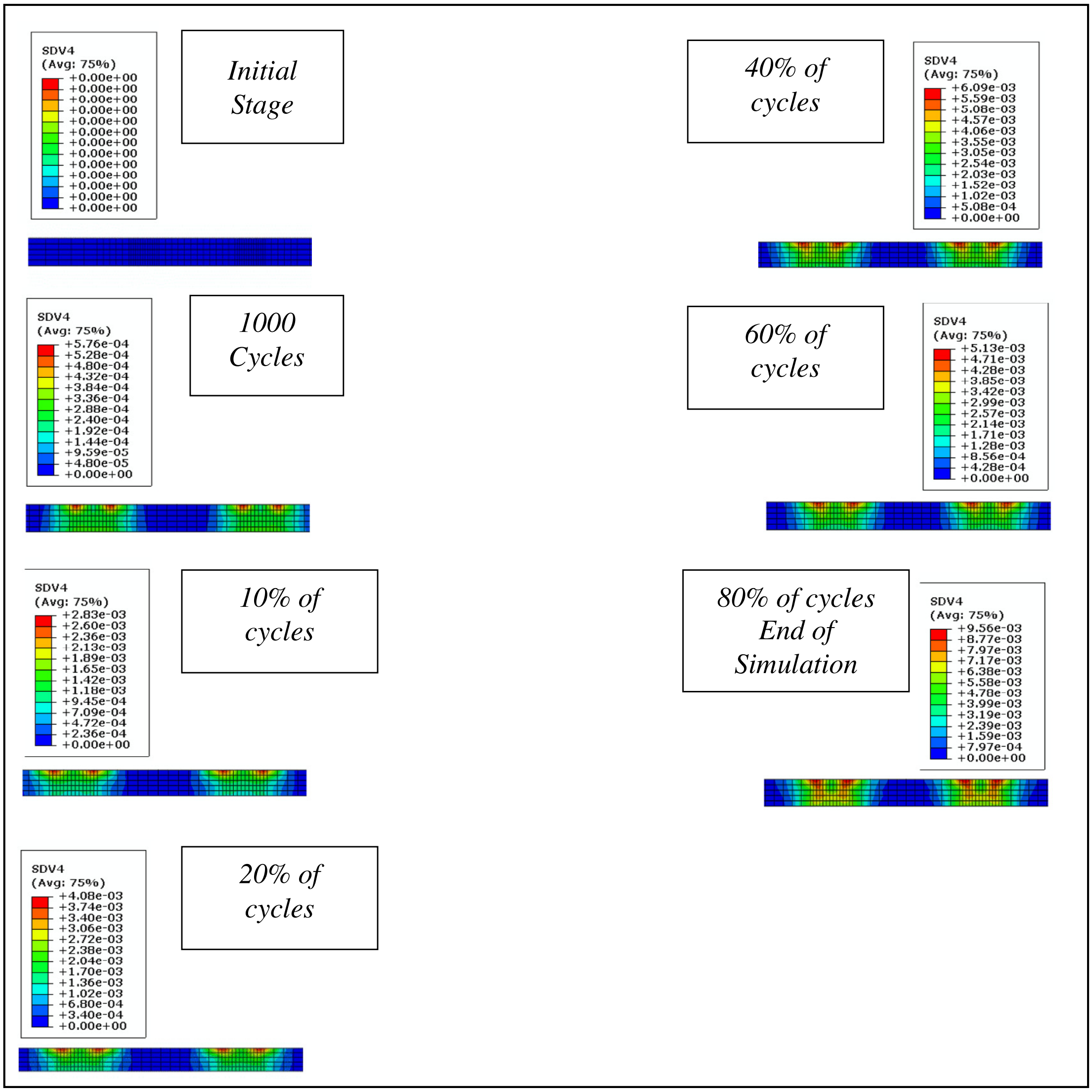

Fig. 9. Sequence of equivalent plastic strain development without shakedown.

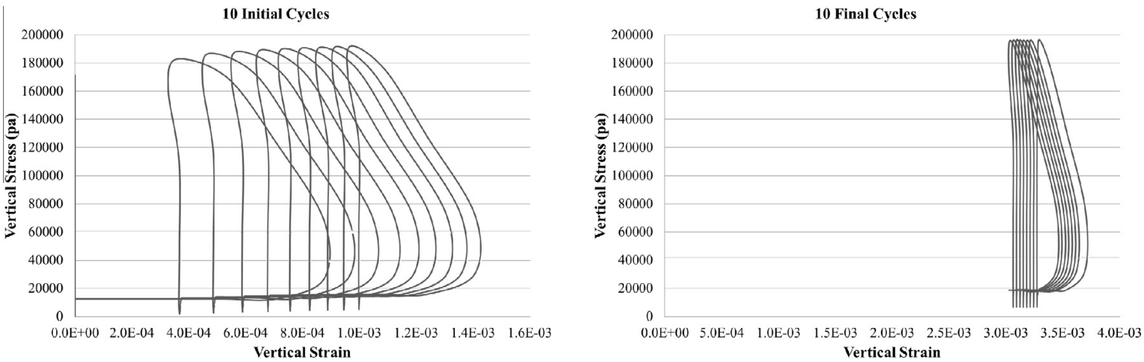

Fig. 10. Initial cycle (left) and final cycle (right) hysteresis loops without the shakedown.

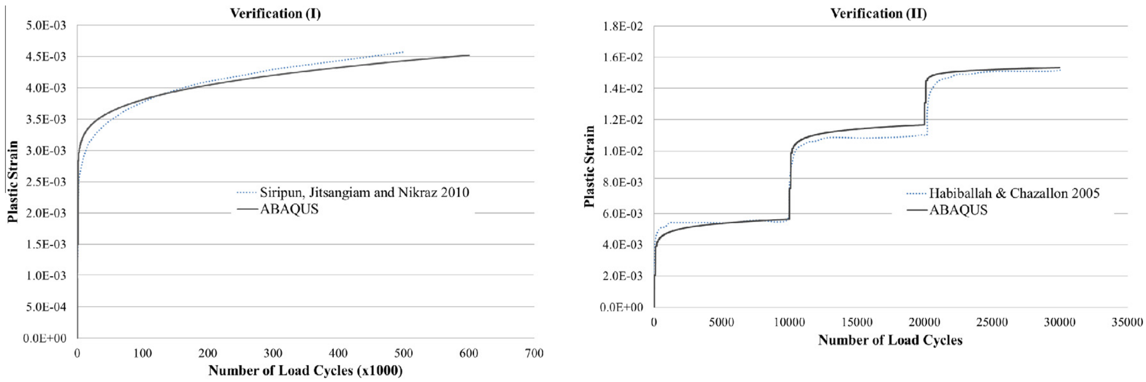

Fig. 11. Verification of the shakedown model.

Another comparison has been made between the experimental results by Habiballah and Chazallon (2005) and the current methodology. The material properties stated in Table 3 of their research is adopted in this model to replicate the growth of plastic strain in repeated triaxial cyclic loading. In this experiment three different values of deviator (q) and confining pressure (p) are selected such that the ratio of q/p is always equal to 3. Fig. 11 illustrates the results of the numerical simulations in the current study versus the experimental results published by Habiballah and Chazallon (2005). The results of their experiments closely follow the results of this study with no more than 5% difference (at 20,000 cycles).

Based on the presented calculations in Fig. 11, it can be said that the proposed methodology in Section Constitutive modelling of materials does successfully simulate the shakedown response of granular materials across different stress states and can successfully simulate dynamic response of granular materials in the base layer of flexible pavements whilst shakedown occurs.

Long term shakedown effects

In this section, the same model used in Section Effect of dynamic analysis is utilised. The material properties are the same as stated in Table 3. A decay function as reported by Siripun et al. (2010) is adopted to replicate the growth of plastic strain in granular layers depending on stress states.

Fig. 12 presents the calculated responses of the layered pavement in sequences of pulse loading and rest period in the first 1000 cycles. It can be seen that the results of the deflection are close to those calculated in Section Effect of dynamic analysis where the maximum deflection is 0.4 mm with a residual of 0.015 mm. This shows that the shakedown did not played a significant role in the response of the pavement during the first 1000 cycles of loading.

However, the effect of shakedown is more noticeable when the long term response of the pavement is considered. Fig. 13 shows the development of vertical deformation in long term dynamic loading. Here the accumulation of vertical deformation does not linearly increase like in Section Effect of dynamic analysis. In this simulation the rate of accumulation of vertical deformation decreased by the number of loading cycles. The accumulated deformation increased from 4E−4 m to 5.8E−4 m at the end of the dynamic simulation. This can be compared with 10E−4 m vertical deformation when shakedown effects are not taken into account.

The effect of shakedown can also be studied on the gradual growth of vertical strain. Fig. 13 illustrates the vertical strain at the top of the base layers. In this figure it can be seen that vertical strain has increased from 0.000582 to 0.000682. This is 18.4% of the value that has been calculated as vertical strain when shakedown is not considered. It can also be observed that the growth of vertical strain is very small at the end of the dynamic analysis.

The results of accumulated equivalent plastic strain are demonstrated in Fig. 13. It can be seen that the equivalent plastic strain is less than 25% of the calculated value when shakedown effects are not considered.

The contours of development of equivalent plastic strain in the base layer during the different cycles of loading are illustrated in Fig. 14. The plastic strain developed initially at the top of the base layer, but gradually moved downward to the bottom of the base layer. This can be understood given the compaction of granular materials in the loading cycles.

The hysteresis loops of stress–strain are presented in Fig. 15. It can be seen that in the initial cycles of loading, the hysteresis loops move from the smaller to the larger strain. However, this movement slows significantly in the final loading cycles. It appears that the hysteresis loops are repeating each other in the same path. This is similar to the behaviour presented in case B of Fig. 3.

The results of the four critical pavement responses of this simulation are presented in Table 4. All values increased from the initial to the final cycle except for vertical stress at the top of subgrade which decreased. This is due to the compaction of the base layer which resulted in less stress being induced in the subgrade layer.

Effect of soil-asphalt interaction

In this section the effects of SAI is investigated. Since the asphalt layer is modelled as a linear elastic material,

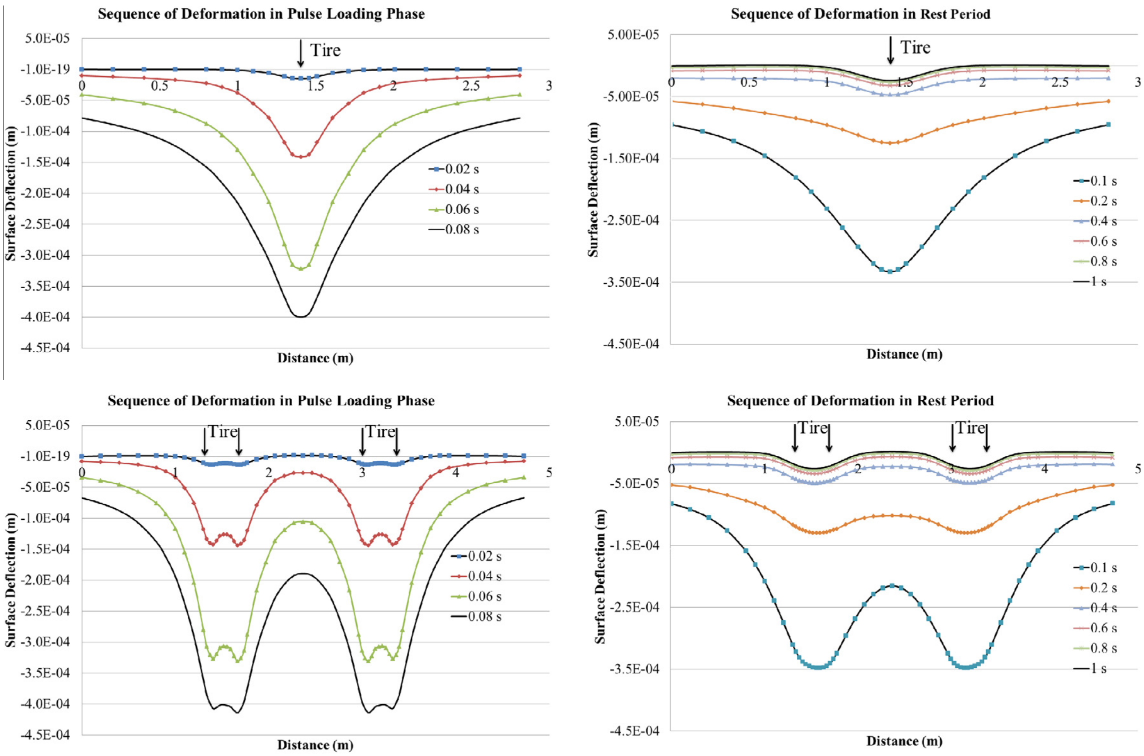

Fig. 12. Longitudinal (up) and transverse (bottom) deformation with the shakedown.

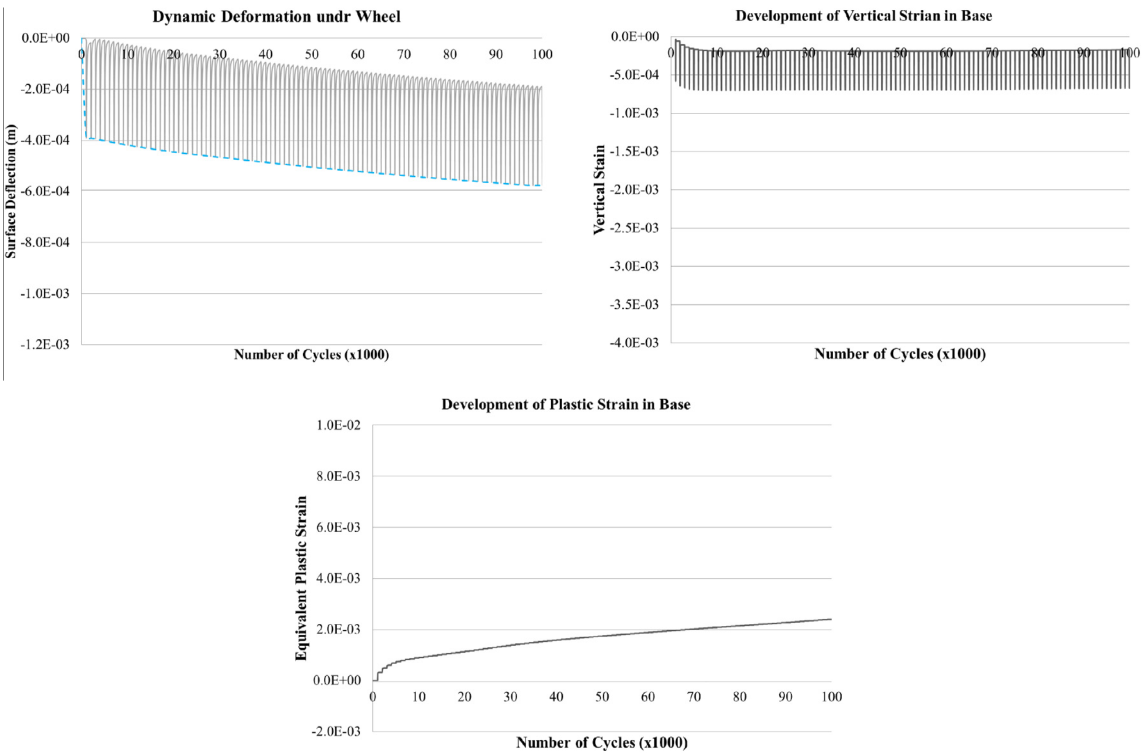

Fig. 13. Mechanical responses of the base beneath the tyre with the shakedown.

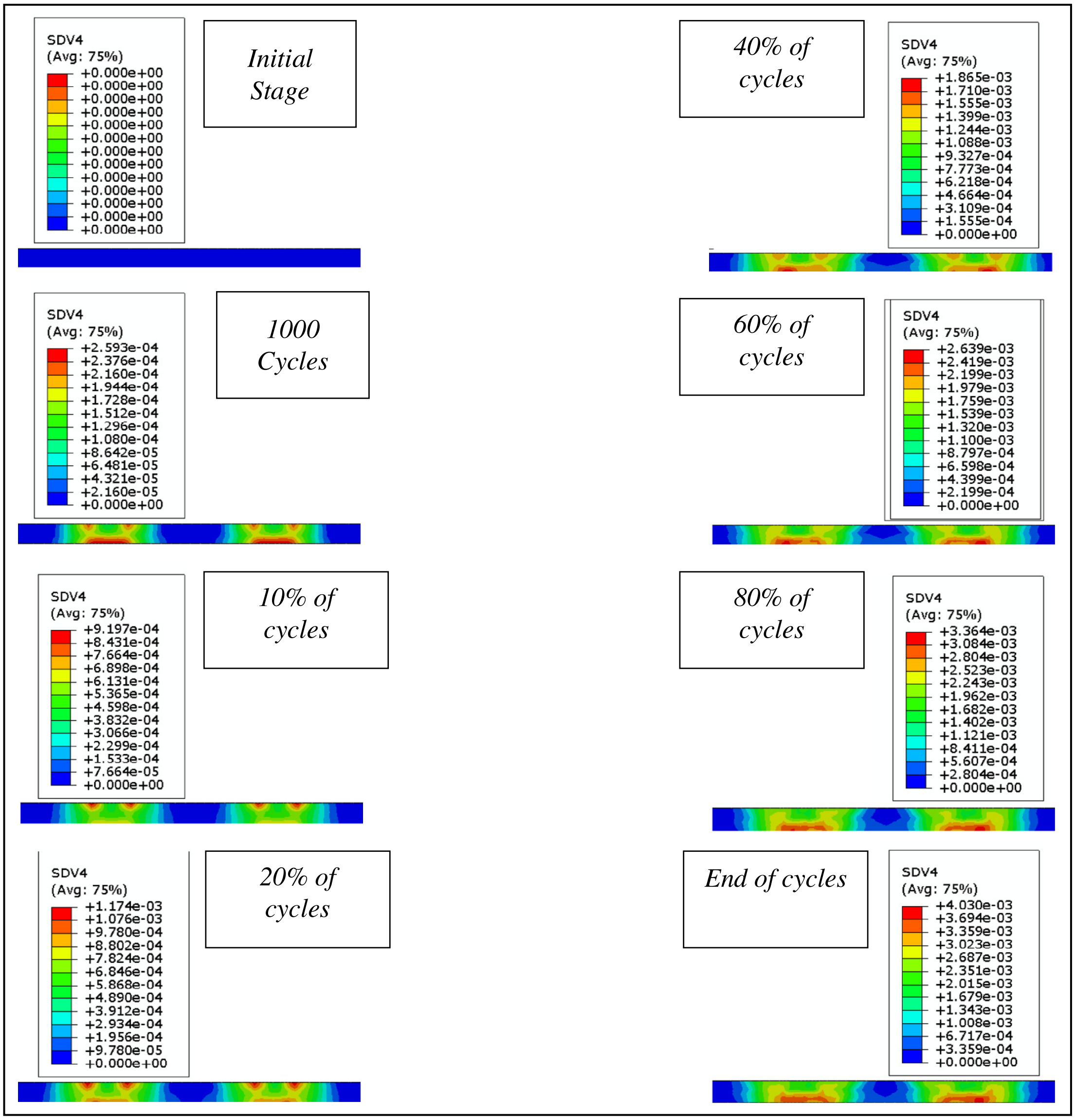

Fig. 14. Sequence of equivalent plastic strain development with shakedown.

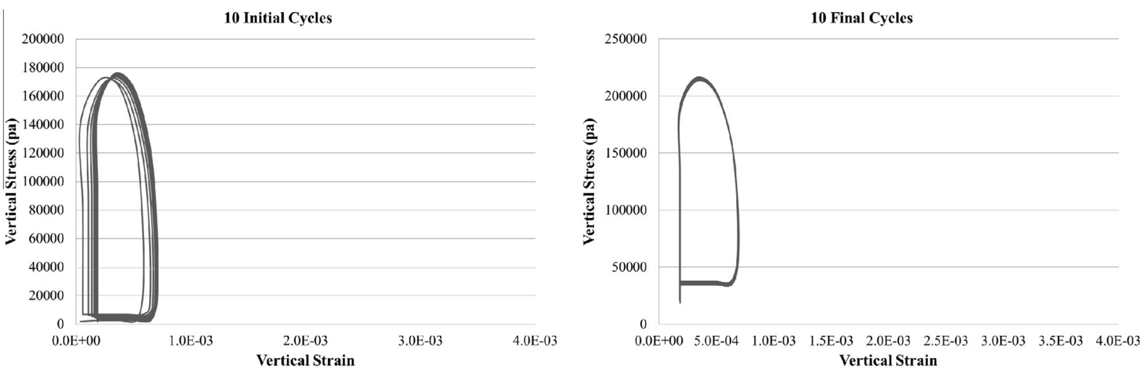

Fig. 15. Initial cycles (left) and final cycles (right) hysteresis loops with shakedown.

Table 4
Reponses of the flexible layer in dynamic analysis (Model 2).

Time	Vertical deflection (top of AC)	Tensile strain (bottom of AC)	Compressive strain (top of SG)	Compressive stress (top of SG)
First 1000th cycles	0.4 mm	168E−6	440E−6	30.8 kPa
Final	0.58 mm	114E−6	480E−6	18.8 kPa

and the structural element, and the elastoplastic granular base layer is the soil element. The interaction is modelled through interface elements which are considered hard contact in the normal direction, and frictional in tangential direction.

The vertical deformation in pulse loading and rest period is illustrated in Fig. 16. Comparing Figs. 16 and 7, a slight decrease in the vertical deformation can be seen when the interaction effects are considered.

In Fig. 17 the accumulation of vertical deformation is presented. The increasing trend is smaller when compared with those presented in Figs. 8 and 13. In this figure vertical deformation increased from 3.75E−4 m to 5.8E−4 m. The effect of dissipation between the base and the asphalt layer can be attributed to the reduction of vertical deformation in this simulation.

The gradual increase of vertical strain and equivalent plastic strain is illustrated in Fig. 17. The vertical strain increased from 0.000265 to 0.000310. This is less than 50% of the vertical strain calculated from simulation in the previous section (0.000682). The interaction between the base and the asphalt has also reduced the development of equivalent plastic strain in comparison with previous simulation.

The progress of equivalent plastic strain is illustrated in Fig. 18. It can be seen that the strain is mainly distributed at the top of the base layer during the simulation period. It is different to the trends in the previous section which note that the plastic strain developed from the top to the bottom of the layer. It can be seen that the location of plastic strain is restricted and cannot progress significantly down the bottom parts of base layer.

The hysteresis loops of stress–strain in this simulation are presented in Fig. 19. In comparison with Fig. 15, less area is enclosed in the hysteresis loops in this simulation. This suggests the behaviour is similar to the elastic shakedown noted in case 3 (Fig. 3).

In Table 5 the four critical responses of simulated pavement structure are presented with the effects of shakedown and SAI. The calculated values in this section are less than the previous section. For example, in this section, vertical strain reached to 360E−6 at the end of the analysis. This value is comparable with 480 E−6 calculated in the previous section.

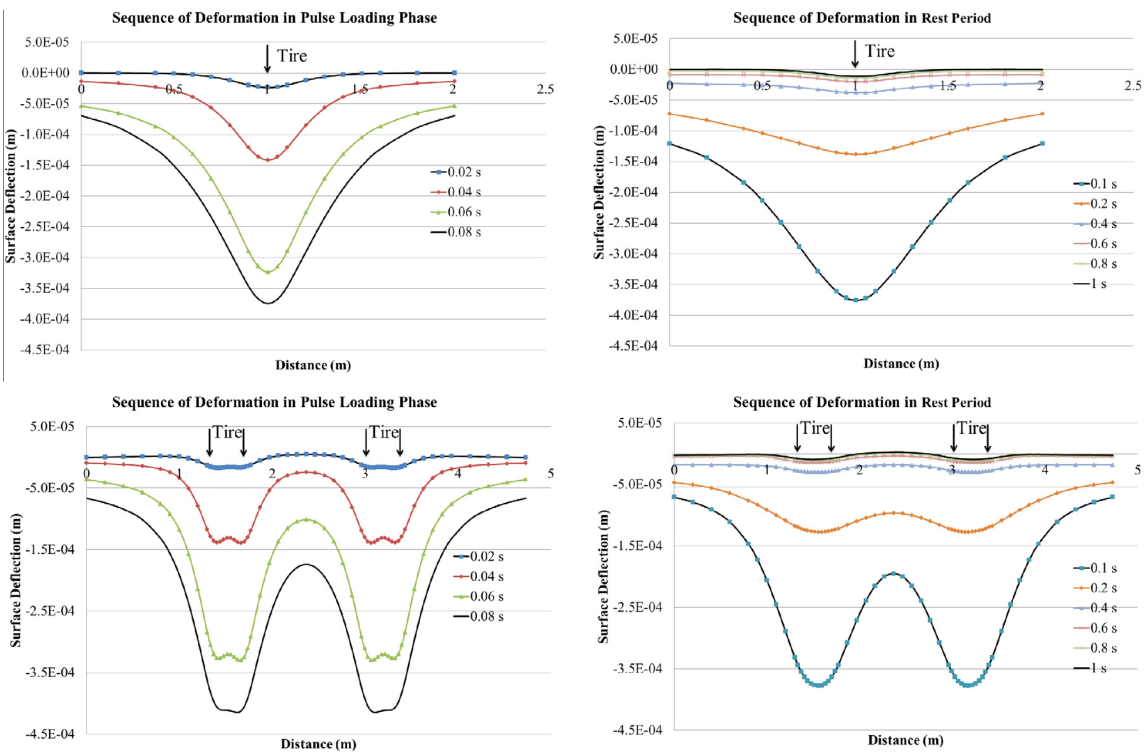

Fig. 16. Longitudinal (up) and transverse (bottom) deformation with SAI.

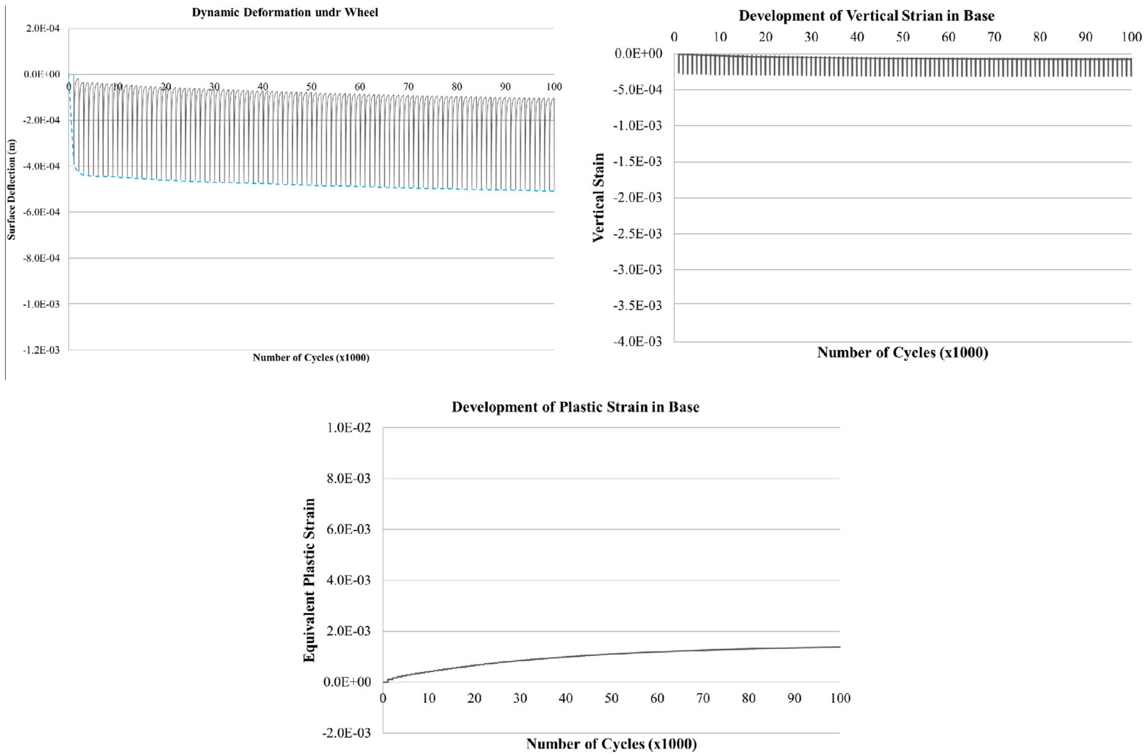

Fig. 17. Mechanical responses of the base beneath tyre with SAI.

Discussion

A comparison between all three different simulations can lead to a better understanding of the mechanical responses of granular materials under stress in pavement layers. It should be remembered that in the first simulation, the base layer behaviour is modelled by the Mohr–Coulomb constitutive equation. In the second simulation the Mohr–Coulomb equation is modified in each loading cycle to replicate shakedown effects. In the third simulation, the effects of SAI are also considered in dynamic simulation.

The developed permanent deformation in the three simulations is presented in Fig. 20. It can be seen that the simulation with SAI, has the least surface deformation at the end of analysis. The final value of surface deformation in simulation 3 is less than half of the calculated value in the first simulation. Therefore, it can be stated that the shakedown and SAI have both reduced the long term surface deformation of the pavement layers significantly.

The same trend can be seen in the development of vertical strain and the equivalent plastic strain in the base layer. For example the final value of the equivalent plastic strain in the third simulation is less than 10% of what is calculated in the first simulation.

In Fig. 21, the final hysteresis loops of the three simulations are presented. A comparison between this figure and Fig. 3 indicates similar responses. In reference to these two figures, the first simulation is similar to case A, the second simulation is similar to case B and the third simulation is identical to case C. In comparison with the concept introduced by Collins and Boulbibane (2000), it can be said that the first simulation reflects "ratcheting behaviour", the second simulation reflects "plastic shakedown" and the third simulation leads to "elastic shakedown".

In Table 6 a comparative percentage of the four critical responses are presented. The values for the three different dynamic simulations are normalised by the calculated values in the static simulation. It can be seen, that the simple Mohr–Coulomb assumption (simulation 1) leads to responses close or even higher than the static analysis at the end of the dynamic analysis. On the other hand, a meaningful reduction in these responses can be seen when shakedown (simulation 2) and SAI (simulation 3) are considered. It should be mentioned that these four critical values are important in flexible pavement design codes (AASHTO, 1993; AUSTROADS, 2004). In these codes, the values of these responses are calculated in a very simple static linear elastic analysis and then, other complex effects are implemented through design trial function (also called transfer function). However, with the development of computational technology it will be possible to conduct a complete mechanical analysis for the entire life time of a given pavement structure.

It is also important to note that in simulation 3, the reduction of critical design values can be attributed to the dissipation of the energy due to interfacial forces between the base and the asphalt layers. This is especially important if the concept of shakedown is considered because it can change the predicted failure mechanism of the entire layered structure.

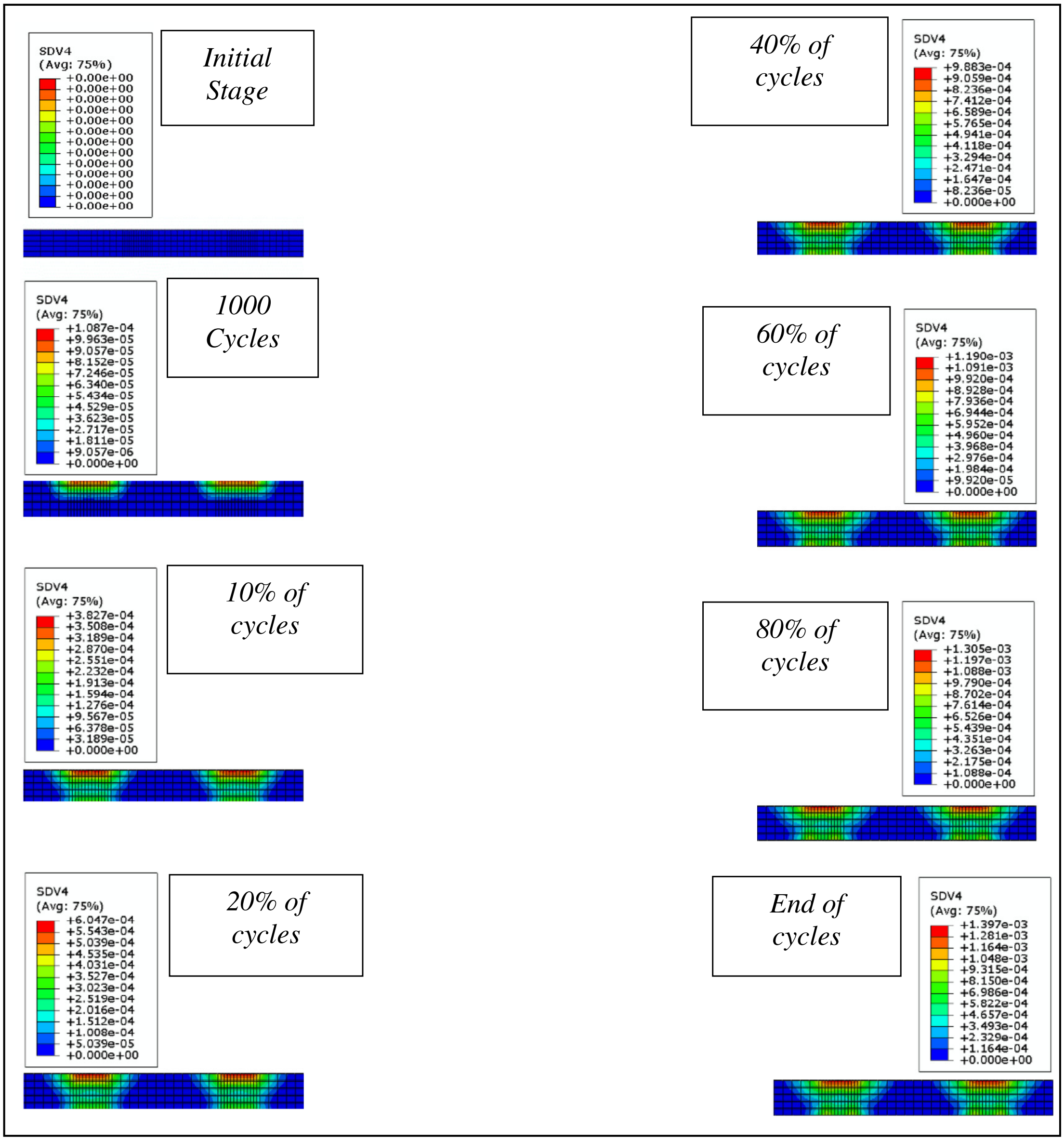

Fig. 18. Sequence of equivalent plastic strain development with SAI.

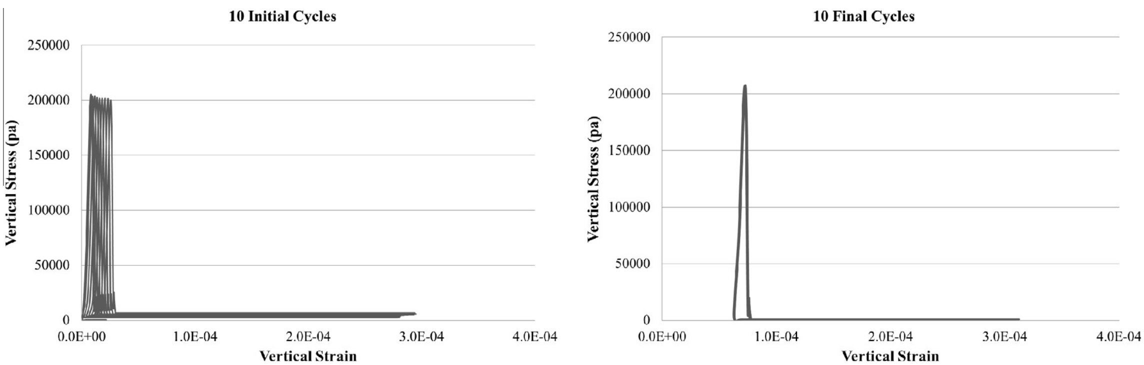

Fig. 19. Initial cycles (left) and final cycles (right) hysteresis loops with SAI.

Table 5
Reponses of the flexible layer in dynamic analysis (Model 3).

Time	Vertical deflection (top of AC)	Tensile strain (bottom of AC)	Compressive strain (top of SG)	Compressive stress (top of SG)
First 1000th cycles	0.14 mm	192E−6	230E−6	27.0 kPa
Final	0.49 mm	201E−6	360E−6	15.5 kPa

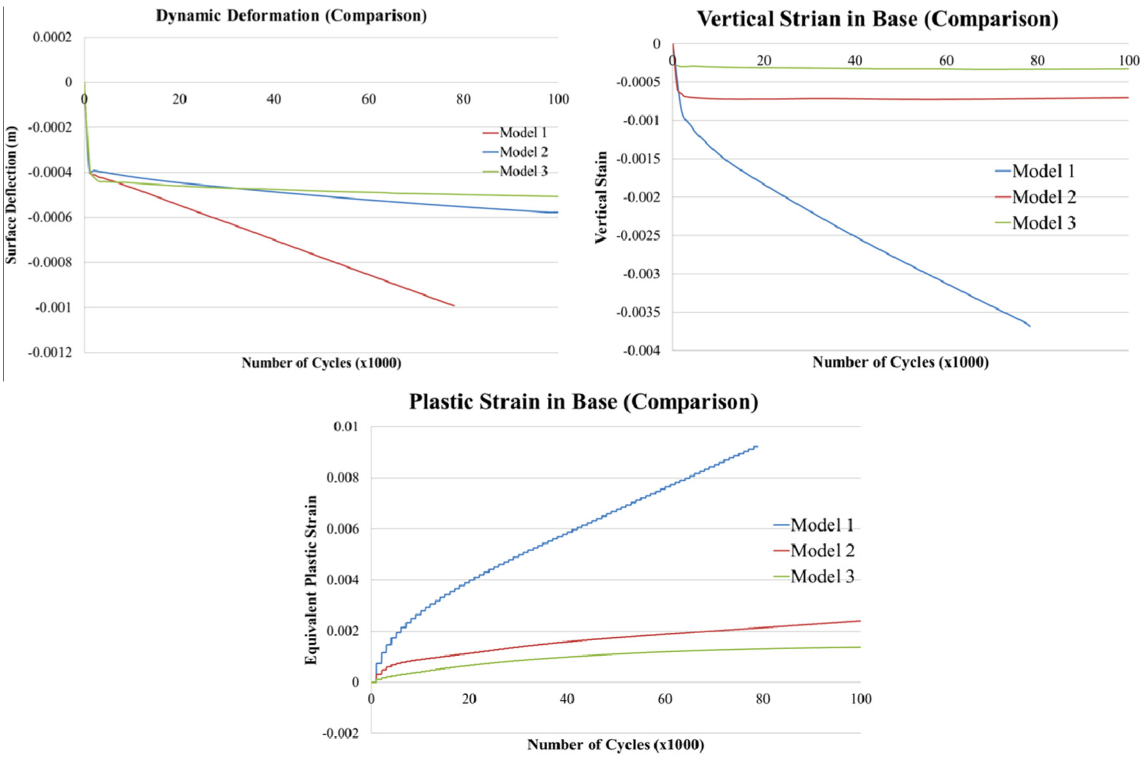

Fig. 20. Comparison of the mechanical responses across three dynamic of simulations.

It should be noted that the effect of shakedown in the compaction of the base layer resulted in less stress transfer to the subgrade layers. Such a finding can be important in the design of road pavement especially when poor subgrade condition is presented.

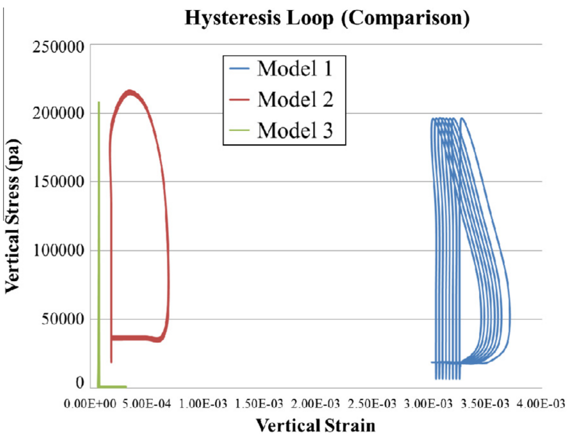

Fig. 21. Comparison of final hysteresis loops in base.

Whilst this research tried to enlighten the significance of dynamic analysis of flexible pavement and SAI effects, it should be noted that the concept of cross-anisotropic behaviour of UGM is not considered in the numerical simulation. It is expected that the cross-anisotropic behaviour of UGM could have a major effect on the total response of layered structure of flexible pavement; however, this was out of scope of current study.

The concept of shakedown is one of the growing research subjects to analyse rutting damage of flexible pavements. Although the behaviour of UGM is complex to model, shakedown is one of the validated responses of UGM under the cyclic loading in laboratory tests (Chazallon et al., 2009; Siripun et al., 2010; Cerni et al., 2012) and field experiments (Chazallon et al., 2009). So far the rutting damage is predicted by combination of mechanistic and empirical approaches. Integration of the shakedown concept in pavement design can provide a fully mechanistic tool to predict rutting damage and optimise the designed asphalt thickness.

Table 6
Comparative percentage of four critical responses across the different simulations.

Time		Vertical deflection (top of AC) (%)	Tensile strain (bottom of AC) (%)	Compressive strain (top of SG) (%)	Compressive stress (top of SG) (%)
Simulation 1	First 1000th cycles	39.60	64.94	23.09	37.69
	Final	99.01	454.55	92.55	113.81
Simulation 2	First 1000th cycles	39.60	72.73	46.81	60.85
	Final	57.43	49.35	51.06	37.14
Simulation 3	First 1000th cycles	37.62	83.12	24.47	53.20
	Final	48.51	87.01	38.30	30.49
Static		100	100	100	100

Conclusion

The finite element simulation of layered flexible pavement is an effective approach in the mechanistic design of road pavement. In this research a comprehensive study on the dynamic simulation of flexible pavement is conducted. Special attention is paid to the mechanical behaviour of granular materials under long term cyclic loading. In this regard, the effects of shakedown and the interactional force between the asphalt and the base layer are considered. Based on this study, the following points can be concluded.

It is observed that compared with static loading, dynamic simulation resulted in lower values in terms of four critical design parameters which are surface deflection under tyre, tensile strain at the bottom of the asphalt layer, compressive stress at the top of subgrade and compressive strain at the top of subgrade. In comparing the first 1000 cycles of dynamic analyses with static analysis it can be stated that firstly, the surface is less than 50% of the static simulation in all of the three dynamic simulations. Secondly, the tensile strain reduces more than 15%. Thirdly, the compressive stress at the top of subgrade is less than 65% of static analysis, and finally the compressive strain at the top of subgrade is less than 50% of static analysis in all simulations.

The interactional forces between asphalt and base can have a significant influence on the final mechanical responses of the layered pavement structure and should not be ignored especially in dynamic simulation. These effects will result in reduction in currently used critical design value in pavement structure and the unnecessary over engineering of pavement thickness.

The shakedown concept can be successfully implemented in FEM simulation through the method introduced in this research. This can have a significant effect on the simulation of the pavement layers under cyclic loading.

In the simulation of a single pavement layer, the assumption of Mohr–Coulomb constitutive models resulted in "ratcheting" behaviour, implementation of the shakedown concept resulted in a "plastic shakedown" response and consideration of SAI resulted in a "elastic shakedown" response. Therefore, it can be said that in a long term dynamic simulation all the effects of shakedown and SAI should be noted to achieve a realistic simulation.

Hysteresis loops of stress–strain in the base layer could be used as an indicator of the mechanical responses of the materials according to the shakedown concepts.

Infinite boundary conditions with sufficient distance from the loading tyre found to be a proper simulation of the realistic conditions.

Inclusion of SAI resulted in higher tensile strain at the bottom of asphalt layer in the dynamic simulations. This strain is the main parameter in the fatigue analysis of the road pavement. Therefore, SAI has an adverse effect on fatigue performance of flexible pavement structure.

Inclusion of SAI resulted in lower compressive strain at the top of subgrade layers. When shakedown effects are also considered, the value decreased further under long term dynamic simulation. Since the compressive strain of the subgrade layer is the leading parameter in rutting failure, it can be said that SAI has a positive effect on the rut performance of the flexible pavement structure.

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